AIAA JOURNAL
Vol. 30, No. 3, March 1992

Scanning Laser Doppler Technique for Modal Testing of Distributed-Parameter Systems

P. Sriram,* Sathya V. Hanagud,† and James I. Craig†
Georgia Institute of Technology, Atlanta, Georgia 30332

Abstract

IN conventional modal testing, accelerometers are used to sense structural response data that are processed to obtain the natural frequencies, damping, and mode shapes of the structure under test. In the case of lightweight structures like composites where mass loading and other local effects of these transducers are not eligible, optical instruments like the laser Doppler vibrometer (LDV) are used. The availability of real-time scanning LDVs has introduced many interesting measurement possibilities. In the method developed in this paper, we process the scanning LDV velocity output signal in the frequency domain to directly obtain the deflection shape of the vibrating structure in a functional (series) form. The technique is demonstrated through application to the problem of modal identification of a cantilever beam. The identified dynamic characteristics of the cantilever beam have been compared with analytical solutions. The method employs a noninvasive, noncontact procedure that avoids the problem of transducer mass loading on lightweight structures and makes possible measurements on hard-to-reach and hazardous surfaces.

Contents

In its basic form, the LDV is essentially a point velocity sensor, with the sensitivity direction being along or perpendicular to the line of sight, depending on the optical configuration. To extend the usefulness of the instrument, manufacturers now offer automatic scanning mechanisms so that the measurements can be performed at a sequence of locations without operator intervention. However, the sensor is stationary during each measurement and the automation has been only in terms of successively moving the sensor to the next measurement point. If we take the concept further along, our system employs a higher scanning rate so that the test data can be considered to be acquired in a simultaneous sense (the sensor is moving continuously while the measurements are in progress). Although high-speed real-time scanning has been reported previously, the data processing has been limited to sampling the scanning LDV output and relating the instaneous velocity to fixed spatial locations, assuming a nonoscillatory velocity field. Using harmonic scanning along a straight line on the surface of a solid test object, our system can be used to obtain the velocity profile along the scan line in a functional form for oscillatory velocity fields also. It is shown that the spatial velocity distribution modulates the velocity output signal from the LDV, and a Fourier analysis of this modulated signal can be used to derive this spatial distribution in a Chebyshev series form. The technique is applicable to systems where the space-time distribution of velocity is of a particular form as described later on. The specific example of a vibrating beam is used as a test case. The beam is assumed to be located in the region $-1 \leq x \leq 1$. The velocity distribution $v(x,t)$ is assumed to be as follows:

$$v(x,t) = g(x) + \phi(x)\sin\omega_b t + \psi(x)\cos\omega_b t \tag{1}$$

where ω_b is the beam vibration frequency. The nonoscillatory spatial velocity distribution $g(x)$ is included for generality whereas $\phi(x)$ and $\psi(x)$ are related to the spatial displacement distribution in the beam (or operating deflection shape). A similar form of space-time velocity distribution occurs in a wide class of vibration problems as well as in fluid flows with periodic structure (e.g., rotor flowfields). The generalized problem posed is to be able to detect $g(x)$, $\phi(x)$, and $\psi(x)$ using the scanning LDV. An LDV scanning at a frequency ω_m, with its sensing location described by

$$x = \cos\omega_m t \tag{2}$$

combines the spatial and temporal velocity variations and the velocity output is governed by

$$\begin{aligned} V(t) &= g(\cos\omega_m t) + \phi(\cos\omega_m t)\sin\omega_b t \\ &\quad + \psi(\cos\psi_{\mathrm{m}}\mathrm{t})\cos\omega_b t \end{aligned} \tag{3}$$

This expression can be rewritten as

$$\begin{aligned} V(t) &= C_0 + S_0\, e^{j\omega_b t} + S_0^* e^{-j\omega_b t} \\ &\quad + \frac{1}{2}\sum_{k=1}^{\infty} (C_k e^{jk\omega_m t} + C_k^* e^{-jk\omega_m t} \\ &\quad + S_k e^{j(\omega_b \pm k\omega_{\mathrm{m}}\mathrm{t})} + S_k^* e^{-j(\omega_b \pm k\omega_m t)}) \end{aligned} \tag{4}$$

where the asterisk represents complex conjugate and S_k is defined as

$$S_k = ½\ (B_k - jA_k) \tag{5}$$

Here, A_k, B_k, and C_k are the Chebyshev series expansion coefficients of $\phi(x)$, $\psi(x)$, and $g(x)$, respectively. For example, we have

$$g(x) = C_0 + \sum_{k=1}^{\infty} C_k T_k(x) \tag{6}$$

where $T_k(x)$ is the kth Chebyshev polynomial. From Eq. (4), it is evident that a Fourier transform of the scanning LDV velocity output signal will exhibit peaks at frequencies 0 (dc), $k\omega_m$, ω_b, and $\omega_b \pm k\omega_m$, with the amplitudes dependent on the nature of g, ϕ, and ψ. Measurement of the complex amplitudes at these frequencies will therefore yield the coefficients A_k, B_k, and C_k that determine the distributions $g(x)$, $\phi(x)$, and $\psi(x)$, and hence $V(x,t)$. The locations of these peaks in the frequency domain are illustrated in Fig. 1. If discrete Fourier analysis (e.g., FFT) is used, it is essential to perform the Fourier transform such that the spectral lines of interest do not overlap. As illustrated in the figure, if the scan rate is set to a $1/(n+0.5)$ fraction of the beam vibration frequency for

Presented as Paper 89-1292 at the AIAA/ASME/ASCE/AHS/ASC 30th Structures, Structural Dynamics, and Materials Conference, Mobile, AL, April 3-5, 1989; received Aug. 13, 1990; synoptic received April 9, 1991; accepted for publication April 21, 1991. Full paper available from AIAA Library, 555 W. 57th Street, New York, NY 10019.

*Post Doctoral Fellow, School of Aerospace Engineering; currently, Assistant Professor, Department of Aerospace Engineering, Wichita State Univ., Wichita, KS 67208.

†Professor, School of Aerospace Engineering.

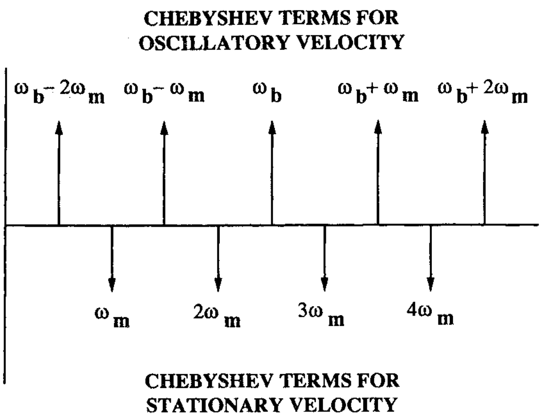

Fig. 1 Frequency components of LDV velocity distribution.

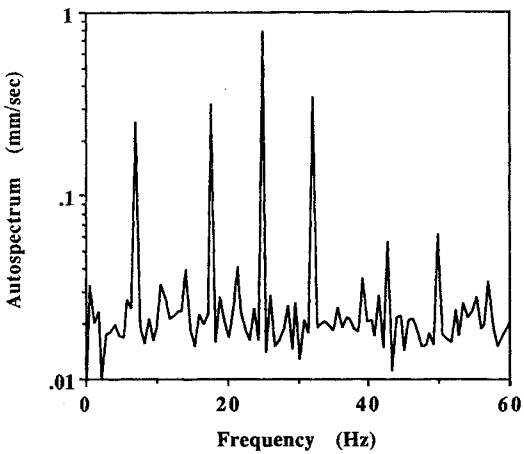

Fig. 2 First mode scanning LDV output autospectrum for beam.

some integer n, the lines are distinct and no overlap occurs. The special case where the velocity is stationary [only $g(x)$] has been dealt with previously,[1] and in this paper we consider the problem of oscillatory velocity measurement.

As a test case, an acrylic plastic rectangular bar of cross section 26 × 2.8 mm was clamped between two aluminum blocks to produce a cantilever beam of length 130 mm. This beam was excited at its first natural frequency of 25 Hz by attaching an electrodynamic shaker at the tip, and the LDV was set to scan the entire length of the beam at $\omega_m = 7.143$ Hz.

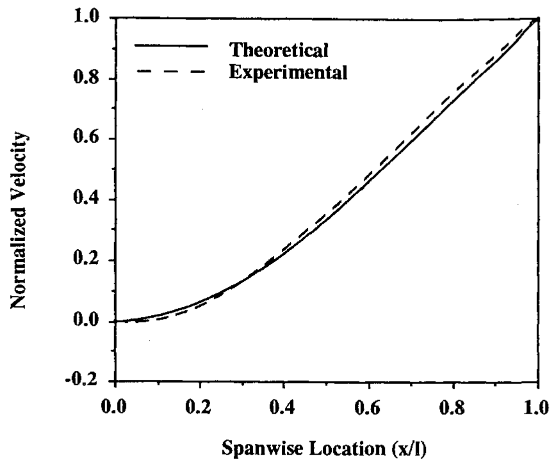

Fig. 3 Cantilever beam first mode measurement using four-term approximation.

The autospectrum of the scanning LDV output, averaged using 10 blocks of 256 samples, is presented in Fig. 2. The first large peak in the autospectrum corresponds to the fundamental scan (ω_m) harmonic. The largest peak in the figure is at the beam exitation frequency ($\omega_b = 25$ Hz) and the two large peaks on either side (the second highest and third highest peaks in the figure) correspond to the $\omega_b \pm \omega_m$ terms. Further terms in the series are difficult to readily observe in the plot, although several are visible.

The Chebyshev coefficients A_0 to A_3 and B_0 to B_3 have been estimated based on the transforms from the scanning LDV data. The coefficients were averaged using an error-minimization procedure. The spatial velocity distribution obtained from these Chebyshev coefficients is presented in Fig. 3. Assuming light damping and wide separation of the natural frequencies of the beam, this spatial distribution is an approximation to the corresponding beam mode shape. Comparison with the theoretical mode shape plotted in Fig. 3 establishes the viability of the measurement technique. The result is especially encouraging considering the fact that the velocity resolution was about 0.0004 m/s and the largest Chebyshev coefficient was only about 0.03 m/s.

Reference

[1]Sriram, P., Hanagud, S., Craig, J. I., and Komerath, N. M., "A New Laser Doppler Technique for Velocity Profile Sensing," *Applied Optics*, Vol. 29, No. 16, 1990, pp. 2409–2417.